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Dualisation of the $D = 7$ heterotic string

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Abstract: The dualisation and the first-order formulation of the $D = 7$ abelian Yang-Mills supergravity which is the low energy effective limit of the $D = 7$ fully Higssed heterotic string is discussed. The non-linear coset formulation of the scalars is enlarged to include the entire bosonic sector by introducing dual fields and by constructing the Lie superalgebra which generates the dualized coset element.

Keywords: Duality in Gauge Field Theories, Supersymmetry and Duality, Supergravity Models, Supersymmetric Effective Theories.

Contents

1. Introduction

The supergravity theory which has the highest spacetime dimension is the $D = 11$, $N = 1$ supergravity [1]. There are three types of supergravity theories in ten-dimensions namely the IIA, [2, 3, 4], the IIB, [5, 6, 7] and as the third supergravity, the ten-dimensional type-I supergravity theory which is coupled to the Yang-Mills theory [8, 9]. One can obtain the $D = 10$, IIA supergravity theory by the Kaluza-Klein dimensional reduction of the $D = 11$ supergravity on the circle, S^1. The supergravity theories for $D < 10$ dimensions can be obtained from the $D = 11$ and the $D = 10$ supergravities by the dimensional reduction and the truncation of fields. The ten dimensional IIA and the IIB supergravity theories are the massless sectors or the low energy effective limits of the type IIA and the type-IIB superstring theories respectively. The type-I Yang-Mills supergravity theory in ten-dimensions on the other hand is the low energy effective limit of the type-I superstring theory and the heterotic string theory. The eleven dimensional supergravity is conjectured to be the low energy effective theory of the eleven dimensional M theory.

The global symmetries of the supergravities which are obtained from the $D = 11$ supergravity [1] by torodial compactification and by partial dualisation of the fields, also the coset formulation of the scalar sectors of these theories are studied in [10]. The complete dualisation and thus the non-linear realization of the bosonic sectors of the IIB and the maximal supergravities which can be obtained by the Kaluza-Klein dimensional reduction of the $D = 11$ supergravity [1] over the tori T^n are given in [11]. These supergravities have scalar coset manifolds G/K which are based on split real form global symmetry groups G [12, 13, 14]. The dualisation of a generic scalar coset which has a split real form global symmetry group is studied in [14] whereas this formulation is generalized to the non-split scalar cosets in [15]. Based on the analysis of [15], the non-linear realization of the $D = 8$ Salam-Sezgin matter coupled supergravity which has a non-split scalar coset is studied in [16]. A general dualisation treatment of the matter coupled non-split scalar cosets is also given [17].

The low energy effective limit of the seven dimensional heterotic string theory is the $D = 7$ supergravity [18, 19] which is coupled to 19 vector supermultiplets [20]. The $D = 7$ matter coupled supergravity is constructed in [21] for an arbitrary number of vector supermultiplets. In this work we perform the complete bosonic dualisation of the $D = 7$ matter coupled supergravity which is an abelian Yang-Mills supergravity [21] and which corresponds to the low energy effective limit of the fully Higgsed $D = 7$ heterotic string. Likewise in [11] the dualisation of the bosonic fields will lead to the construction of the bosonic sector of the $D = 7$ abelian Yang-Mills supergravity as a non-linear coset model and we will also obtain the first-order formulation of the theory.

In section two we will discuss the coset formulation of the scalar sector of the $D = 7$ matter coupled supergravity [21] also after deriving the bosonic field equations we will locally integrate them to obtain the first-order field equations. In section three by following the dualisation method of [11] we will introduce dual fields (Lagrange multipliers [22]) for the bosonic field content and construct the Lie superalgebra which generates the dualized coset element whose Cartan form realizes the original second-order field equations by satisfying the Cartan-Maurer equation and the first-order field equations by satisfying a twisted self-duality condition [11, 14, 15, 16, 17]. Since the scalar coset of the $D = 7$ abelian Yang-Mills supergravity is in general a non-split type depending on N (the number of the coupling vector multiplets) we will effectively use the results of [15] which studies the dualisation of the non-split scalar cosets.

2. The effective $D = 7$ heterotic string

The matter coupled $D = 7$ supergravity can be obtained [23] by the T^3-compactification of the $D = 10$ type-I supergravity that is coupled to the Yang-Mills theory [8, 9]. The ten dimensional type-I supergravity which is coupled to 16 vector supermultiplets is the low energy effective limit of the $D = 10$ heterotic string. Thus the $D = 7$ matter coupled Yang-Mills supergravity [21] is the low energy effective limit of the $D = 7$ heterotic string theory [20]. We will consider the fully Higgsed vacuum configuration for the heterotic string which causes a spontaneous symmetry breakdown of the full Yang-Mills gauge symmetry to its maximal torus subgroup. The low energy effective limit of the fully Higgsed $D = 10$ heterotic string is the $\mathrm{U}(1)^{16}$ abelian Yang-Mills supergravity where the $E_8 \times E_8$ gauge symmetry is replaced by its maximal torus subgroup $\mathrm{U}(1)^{16}$ whose Lie algebra is the Cartan subalgebra of $E_8 \times E_8$. For the lower dimensional theories one may consider the dimensional reduction of the $D = 10$ abelian Yang-Mills supergravity which is the maximal torus subtheory of the $D = 10$ Yang-Mills supergravity [23]. Therefore the $D = 7$ abelian Yang-Mills supergravity [21] corresponds to the low energy effective limit of the fully Higgsed $D = 7$ heterotic string. We will follow a general formulation and consider an arbitrary number N of coupling vector multiplets bearing in mind that if one chooses $N = 19$ one obtains the effective fully Higssed $D = 7$ heterotic string.

The original field content of the seven dimensional pure $\mathcal{N} = 2$ supergravity consists of a siebenbein e_μ^m, $\mathrm{Sp}(1)$ pseudo-Majorana gravitinos ψ_μ^i for $i = 1, 2$, an $\mathrm{Sp}(1)$ pseudo-Majorana spinor χ^k, for $j = 1, 2, 3$ a triplet of one-forms $\{A_\mu^j\}$, a two-form field $B_{\mu\nu}$ and a dilaton σ. In [21] the $D = 7$ abelian Yang-Mills supergravity is constructed by coupling to the original fields an arbitrary number of N vector multiplets

$$(A_\mu, \lambda^i, \varphi^\alpha)\,, \tag{2.1}$$

where A_μ is a one-form field, λ^i is an Sp(1) pseudo-Majorana spinor and for $\alpha = 1, 2, 3$ the scalars φ^α form an Sp(1) triplet. Thus the total field content of the seven dimensional abelian Yang-Mills supergravity is

$$\left(e_\mu^m, \psi_\mu^i, \chi^k, B_{\mu\nu}, \sigma, A_\mu^I, \lambda^{ai}, \varphi^\beta\right), \tag{2.2}$$

where $I = 1, \ldots, N + 3$, $a = 1, \ldots, N$ and $\beta = 1, \ldots, 3N$. The $3N$ scalars φ^β of the N vector multiplets parameterize the $\mathrm{SO}(N, 3)/\mathrm{SO}(N) \times \mathrm{SO}(3)$ coset manifold. $\mathrm{SO}(N, 3)$ is in general a non-compact real form of a semi-simple Lie group and $\mathrm{SO}(N) \times \mathrm{SO}(3)$ is its maximal compact subgroup. For this reason $\mathrm{SO}(N, 3)/\mathrm{SO}(N) \times \mathrm{SO}(3)$ is a riemannian globally symmetric space for all the $\mathrm{SO}(N, 3)$-invariant riemannian structures on $\mathrm{SO}(N, 3)/\mathrm{SO}(N) \times \mathrm{SO}(3)$ [12]. Depending on the number of coupling vector multiplets $\mathrm{SO}(N, 3)$ can be a maximally non-compact (split) real form or not. We will consider the general case of non-split real forms for $\mathrm{SO}(N, 3)$ which contains the split real forms as special cases. Therefore the part of the bosonic lagrangian which governs the vector multiplet scalars φ^β can be expressed as a general symmetric space sigma model lagrangian [15]. One can use the solvable Lie algebra parametrization [24] to parameterize the scalar coset manifold $\mathrm{SO}(N, 3)/\mathrm{SO}(N) \times \mathrm{SO}(3)$. The solvable Lie algebra parametrization is based on the Iwasawa decomposition [12]

$$\begin{aligned}\mathfrak{so}(N, 3) &= \mathbf{k}_0 \oplus \mathbf{s}_0 \\ &= \mathbf{k}_0 \oplus \mathbf{h_k} \oplus \mathbf{n_k}\,,\end{aligned} \tag{2.3}$$

where $\mathbf{k}_0$ is the Lie algebra of $\mathrm{SO}(N) \times \mathrm{SO}(3)$ and $\mathbf{s}_0$ is the solvable Lie subalgebra of $\mathfrak{so}(N, 3)$. In (2.3) $\mathbf{h_k}$ is a subalgebra of the Cartan subalgebra $\mathbf{h}_0$ of $\mathfrak{so}(N, 3)$ and it generates the maximal R-split torus in $\mathrm{SO}(N, 3)$ [12, 15, 24, 25]. The nilpotent Lie subalgebra $\mathbf{n_k}$ of $\mathfrak{so}(N, 3)$ is generated by a subset $\{E_m\}$ of the positive root generators of $\mathfrak{so}(N, 3)$ such that $m \in \Delta_{nc}^+$. The roots in Δ_{nc}^+ are the non-compact roots with respect to the Cartan involution θ which is induced by the Cartan decomposition [12, 15, 17]

$$\mathfrak{so}(N, 3) = \mathbf{k}_0 \oplus \mathbf{u}_0\,. \tag{2.4}$$

The difeomorphism from $\mathbf{u}_0 \simeq \mathbf{s}_0$ onto the riemannian globally symmetric space $\mathrm{SO}(N, 3)/\mathrm{SO}(N) \times \mathrm{SO}(3)$ [12] enables the construction of the solvable Lie algebra parametrization for $\mathrm{SO}(N, 3)/\mathrm{SO}(N) \times \mathrm{SO}(3)$ such that the coset representatives can be expressed as

$$\begin{aligned}L &= \mathbf{g}_H \mathbf{g}_N \\ &= e^{\frac{1}{2}\phi^i(x) H_i} e^{\chi^m(x) E_m}\,,\end{aligned} \tag{2.5}$$

where $\{H_i\}$ for $i = 1, \ldots, \dim \mathbf{h}_k$ are the generators of $\mathbf{h}_k$ and $\{E_m\}$ for $m \in \Delta^+_{nc}$ are the positive root generators which generate the orthogonal complement of $\mathbf{h}_k$ within the solvable Lie algebra $\mathbf{s}_0$ of $\mathfrak{so}(N,3)$ namely $\mathbf{n_k}$. We have classified the $3N$ vector multiplet scalars φ^β as $\{\phi^i\}$, the dilatons for $i = 1, \ldots, \dim \mathbf{h}_k$ and $\{\chi^m\}$, the axions for $m \in \Delta^+_{nc}$. We will label the roots which are elements of Δ^+_{nc} from 1 to n where n is the dimension of $\mathbf{n}_k$. Also we will refer to the dimension of $\mathbf{h}_k$ as r. Thus we have $r+n = 3N$. Since (2.5) is a map from the seven dimensional spacetime into $\mathrm{SO}(N,3)$ (whose range gives the representatives of the left cosets $\mathrm{SO}(N,3)/\,\mathrm{SO}(N) \times \mathrm{SO}(3)$ in one to one correspondence) it satisfies the defining relations of $\mathrm{SO}(N,3)$;

$$L^T \eta L = \eta\,, \qquad L^{-1} = \eta L^T \eta\,, \tag{2.6}$$

where $\eta = (-,-,-,+,+,+,\ldots)$. The coset representatives (2.5) can be chosen as symmetric matrices in the $(N+3)$-dimensional representation which is evident from the explicit construction of the coset space $\mathrm{SO}(N,3)/\,\mathrm{SO}(N) \times \mathrm{SO}(3)$ [23]. Thus we will take the coset representatives as symmetric matrices; $L^T = L$ however likewise in [16] we will keep on using L^T in our formulation for the sake of accuracy. If we assume a matrix representation for the algebra $\mathfrak{so}(N,3)$ then from (2.5) we have

$$\begin{aligned} \partial_i L \equiv \frac{\partial L}{\partial \phi^i} = \frac{1}{2} H_i L\,, \qquad \partial_i L^T \equiv \frac{\partial L^T}{\partial \phi^i} = \frac{1}{2} L^T H_i^T\,, \\ \partial_i L^{-1} \equiv \frac{\partial L^{-1}}{\partial \phi^i} = -\frac{1}{2} L^{-1} H_i\,. \end{aligned} \tag{2.7}$$

By differentiating the identities (2.6) with respect to the dilatons ϕ^i also by using the identities (2.7) bearing in mind that the generators $\{H_i\}$ are Cartan generators and we can choose the coset representatives (2.5) as symmetric matrices one can show that

$$(H_i L)^T = H_i L\,, \qquad H_i^T \eta = -\eta H_i\,. \tag{2.8}$$

If we introduce the internal metric

$$\mathcal{M} = L^T L\,, \tag{2.9}$$

then the lagrangian which corresponds to the $3N$ vector multiplet scalars of the $D = 7$ abelian Yang-Mills supergravity can be given as [11, 15, 17, 25]

$$\mathcal{L}_{\text{scalar}} = -\frac{1}{8} \operatorname{tr}(*d\mathcal{M}^{-1} \wedge d\mathcal{M})\,. \tag{2.10}$$

Thus now we can express the bosonic lagrangian of the $D = 7$ abelian Yang-Mills supergravity as [21]

$$\begin{aligned} \mathcal{L} = &\frac{1}{2} R * 1 - \frac{5}{8} * d\sigma \wedge d\sigma - \frac{1}{2} e^{2\sigma} * G \wedge G - \\ &- \frac{1}{8} \operatorname{tr}(*d\mathcal{M}^{-1} \wedge d\mathcal{M}) - \frac{1}{2} e^{\sigma} F \wedge \mathcal{M} * F\,, \end{aligned} \tag{2.11}$$

where the coupling between the field strengths $F^I = dA^I$ for $I = 1, \ldots, N+3$ and the scalars which parameterize the coset $\mathrm{SO}(N,3)/\,\mathrm{SO}(N)\times\mathrm{SO}(3)$ can be explicitly written as

$$-\frac{1}{2}e^{\sigma} F \wedge \mathcal{M} * F = -\frac{1}{2}e^{\sigma}\mathcal{M}_{ij}F^i \wedge *F^j \,. \tag{2.12}$$

We have assumed the $(N+3)$-dimensional matrix representation of $\mathfrak{so}(N,3)$. The Chern-Simon form G is defined as [21]

$$G = dB - \frac{1}{\sqrt{2}}\eta_{ij}A^i \wedge F^j \,. \tag{2.13}$$

The lagrangian (2.11) can be identified as the T^3-compactification of the $D = 10$ abelian Yang-Mills supergravity [20, 23] and it has an equivalent dual form in which the two-form potential B is replaced by a Lagrange multiplier three-form field [18, 19]. This dual lagrangian corresponds to the K_3 compactification of the $D = 11$ supergravity [26].

If we vary the lagrangian (2.11) with respect to the fields σ, B and $\{A^i\}$ we can find the corresponding field equations as

$$\begin{aligned} -\frac{5}{4}d(*d\sigma) &= -e^{2\sigma} * G \wedge G - \frac{1}{2}e^{\sigma}\mathcal{M}_{ij} * F^i \wedge F^j \,, \\ d(e^{2\sigma} * G) &= 0 \,, \\ d(e^{\sigma}\mathcal{M}_{ij} * F^j) &= \sqrt{2}\, e^{2\sigma}\eta_{ij}F^j \wedge *G \,. \end{aligned} \tag{2.14}$$

The last equation can also be expressed as

$$\begin{aligned} d(e^{\frac{1}{2}\sigma}L * F) = &-\frac{1}{2}d\sigma \wedge e^{\frac{1}{2}\sigma}L * F - \mathcal{G}_0^T \wedge e^{\frac{1}{2}\sigma}L * F + \\ &+ \sqrt{2}\,(L^T)^{-1}\eta\, e^{\frac{1}{2}\sigma}F \wedge e^{\sigma} * G \,, \end{aligned} \tag{2.15}$$

where we have used the Cartan-Maurer form

$$\mathcal{G}_0 = dLL^{-1} \,, \tag{2.16}$$

which is calculated in [15] as

$$\begin{aligned} \mathcal{G}_0 &= \frac{1}{2}d\phi^i H_i + e^{\frac{1}{2}\alpha_i\phi^i}U^{\alpha}E_{\alpha} \\ &= \frac{1}{2}d\phi^i H_i + \overset{\rightharpoonup}{\mathbf{E}'}\,\mathbf{\Omega}\,\overset{\rightharpoonup}{d\chi} \,, \end{aligned} \tag{2.17}$$

where $\{H_i\}$ for $i = 1, \ldots, r$ are the generators of $\mathbf{h}_k$ and $\{E_\alpha\}$ for $\alpha \in \Delta_{nc}^+$ are the generators of $\mathbf{n}_k$ as defined before. We have introduced the column vector

$$U^{\alpha} = \mathbf{\Omega}^{\alpha}_{\beta}d\chi^{\beta} \,, \tag{2.18}$$

and the row vector

$$\left(\overset{\rightharpoonup}{\mathbf{E}'}\right)_{\alpha} = e^{\frac{1}{2}\alpha_i\phi^i}E_{\alpha} \,, \tag{2.19}$$

where the components of the roots $\alpha \in \Delta_{nc}^{+}$ are defined as $[H_i, E_\alpha] = \alpha_i E_\alpha$. Also $\mathbf{\Omega}$ is an $n \times n$ matrix

$$\begin{aligned}\mathbf{\Omega} &= \sum_{m=0}^{\infty} \frac{\omega^m}{(m+1)!} \\ &= (e^{\omega} - I)\, \omega^{-1} .\end{aligned} \tag{2.20}$$

The matrix ω is $\omega_\beta^\gamma = \chi^\alpha K_{\alpha\beta}^\gamma$. The structure constants $K_{\alpha\beta}^\gamma$ are defined as $[E_\alpha, E_\beta] = K_{\alpha\beta}^\gamma E_\gamma$. In other words since $[E_\alpha, E_\beta] = N_{\alpha,\beta} E_{\alpha+\beta}$ we have $K_{\beta\beta}^\alpha = 0$, $K_{\beta\gamma}^\alpha = N_{\beta,\gamma}$ if $\beta + \gamma = \alpha$ and $K_{\beta\gamma}^\alpha = 0$ if $\beta + \gamma \neq \alpha$ in the root sense.

By following the analysis of [13, 25] the field equations for $\{\phi^i\}$ and $\{\chi^m\}$ can be found as

$$\begin{aligned}d(e^{\frac{1}{2}\gamma_i\phi^i} * U^\gamma) =& -\frac{1}{2}\gamma_j e^{\frac{1}{2}\gamma_i\phi^i} d\phi^j \wedge *U^\gamma + \\ &+ \sum_{\alpha-\beta=-\gamma} e^{\frac{1}{2}\alpha_i\phi^i} e^{\frac{1}{2}\beta_i\phi^i} N_{\alpha,-\beta} U^\alpha \wedge *U^\beta , \\ d(*d\phi^i) =& \frac{1}{2} \sum_{\alpha\in\Delta_{nc}^{+}} \alpha_i e^{\frac{1}{2}\alpha_j\phi^j} U^\alpha \wedge e^{\frac{1}{2}\alpha_j\phi^j} * U^\alpha - \\ & - e^{\sigma}((H_i)_n^a L_m^n L_j^a) * F^m \wedge F^j ,\end{aligned} \tag{2.21}$$

$\alpha, \beta, \gamma \in \Delta_{nc}^{+}$ and $[H_i, E_\alpha] = \alpha_i E_\alpha$ as we have defined above. The matrices $\{(H_i)_n^a\}$ are the ones corresponding to the generators $\{H_i\}$ in the $(N+3)$-dimensional representation chosen.

The field equations corresponding to the scalar lagrangians of the symmetric space sigma models which are in the form of (2.10) for the dilatons $\{\phi^i\}$ and the axions $\{\chi^m\}$ are derived for generic split and non-split scalar cosets G/K in [13, 14, 15, 25]. By referring to [15] the locally integrated first-order field equations for the scalar lagrangian (2.10) can be given as

$$*\vec{\mathbf{\Psi}} = -e^{\mathbf{\Gamma}} e^{\mathbf{\Lambda}} \vec{\mathbf{S}}. \tag{2.22}$$

When one applies the exterior derivative on both sides of (2.22) one finds the second-order field equations of the scalar lagrangian (2.10) [15]. The column vector $\vec{\mathbf{\Psi}}$ is defined as

$$\begin{aligned}\mathbf{\Psi}^i &= \frac{1}{2} d\phi^i \qquad \text{for } i = 1, \ldots, r , \\ \mathbf{\Psi}^{\alpha+r} &= e^{\frac{1}{2}\alpha_i\phi^i} \mathbf{\Omega}_l^\alpha d\chi^l \qquad \text{for } \alpha = 1, \ldots, n ,\end{aligned} \tag{2.23}$$

where we have enumerated the roots in Δ_{nc}^{+}. The vector $\vec{\mathbf{S}}$ is

$$\begin{aligned}S^j &= \frac{1}{2} d\widetilde{\phi}^j \qquad \text{for } j = 1, \ldots, r , \\ S^{\alpha+r} &= d\widetilde{\chi}^\alpha \qquad \text{for } \alpha = 1, \ldots, n ,\end{aligned} \tag{2.24}$$

where $\widetilde{\phi}^j$, $\widetilde{\chi}^\alpha$ are the five-form Lagrange multiplier dual fields [15]. The matrices $\mathbf{\Gamma}$ and $\mathbf{\Lambda}$ are introduced as

$$\mathbf{\Gamma}_n^k = \frac{1}{2} \phi^i \, \widetilde{g}_{in}^k \quad , \quad \mathbf{\Lambda}_n^k = \chi^m \widetilde{f}_{mn}^k , \tag{2.25}$$

where the structure constants $\{\widetilde{g}^k_{in}\}$ and $\{\widetilde{f}^k_{mn}\}$ can be read from

$$[E_\alpha, \widetilde{T}_m] = \widetilde{f}^n_{\alpha m}\widetilde{T}_n\,, \qquad [H_i, \widetilde{T}_m] = \widetilde{g}^n_{im}\widetilde{T}_n\,. \tag{2.26}$$

The dual generators $\widetilde{T}_m$ are defined in [14, 15] as $\widetilde{T}_i = \widetilde{H}_i$ for $i = 1, \ldots, r$ and $\widetilde{T}_{\alpha+r} = \widetilde{E}_\alpha$ for $\alpha = 1, \ldots, n$. Here $\widetilde{H}_i$ and $\widetilde{E}_\alpha$ are the associated dual generators of the dual fields which together with the scalar generators $\{H_i, E_\alpha\}$ parameterize the dualized scalar coset [14, 15]. In [15] the structure constants $\{\widetilde{g}^k_{in}\}$ and $\{\widetilde{f}^k_{mn}\}$ are calculated as

$$\begin{aligned}
\widetilde{f}^n_{\alpha m} &= 0\,, && m \le r\,,\\
\widetilde{f}^i_{\alpha,\alpha+r} &= \frac{1}{4}\alpha_i\,, && i \le r\,,\\
\widetilde{f}^i_{\alpha,\alpha+r} &= 0\,, && i > r\,,\\
\widetilde{f}^i_{\alpha,\beta+r} &= 0\,, && i \le r\,, \quad \alpha \ne \beta\,,\\
\widetilde{f}^{\gamma+r}_{\alpha,\beta+r} &= N_{\alpha,-\beta}\,, && \alpha - \beta = -\gamma\,, \quad \alpha \ne \beta\,,\\
\widetilde{f}^{\gamma+r}_{\alpha,\beta+r} &= 0\,, && \alpha - \beta \ne -\gamma\,, \quad \alpha \ne \beta\,,\\
\widetilde{g}^n_{im} &= 0\,, && m \le r\,,\\
\widetilde{g}^n_{im} &= 0\,, && m > r\,, \quad m \ne n\,,\\
\widetilde{g}^\alpha_{i\alpha} &= -\alpha_i\,, && \alpha > r\,.
\end{aligned} \tag{2.27}$$

By using the first-order formulation of the scalar coset manifold of the $D = 7$ abelian Yang-Mills supergravity namely $\mathrm{SO}(N,3)/\,\mathrm{SO}(N) \times \mathrm{SO}(3)$ given above we can locally integrate the bosonic field equations (2.14) and (2.21). One can introduce dual fields which are nothing but the Lagrange multipliers and by using the fact that locally a closed form is an exact one one can derive the corresponding local first-order field equations such that the exterior derivative operator can be extracted on both sides of the equations (2.14) and (2.21). Thus if we introduce the dual three-form $\widetilde{B}$, the set of dual four-forms $\{\widetilde{A}^j\}$ and the dual five-forms $\{\widetilde{\sigma}, \widetilde{\phi}^i, \widetilde{\chi}^m\}$ we can locally derive the first-order field equations as

$$\begin{aligned}
e^{2\sigma} * G &= -d\widetilde{B}\,,\\
e^{\sigma}\mathcal{M}^i_j * F^j &= -d\widetilde{A}^i - \sqrt{2}\, d\widetilde{B} \wedge \eta^i_j A^j\,,\\
*d\sigma &= -d\widetilde{\sigma} - \frac{4}{5}B \wedge d\widetilde{B} - \frac{2}{5}\delta_{ij}A^i \wedge d\widetilde{A}^j\,,\\
e^{\frac{1}{2}\alpha_i\phi^i}(\mathbf{\Omega})^\alpha_l * d\chi^l &= -(e^{\mathbf{\Gamma}}e^{\mathbf{\Lambda}})^{\alpha+r}_j S^j\,,\\
\frac{1}{2} * d\phi^m &= -(e^{\mathbf{\Gamma}}e^{\mathbf{\Lambda}})^m_j S^j + \frac{1}{2}(H_m)_{ji}A^j \wedge d\widetilde{A}^i + \frac{1}{2\sqrt{2}}\,\eta^k_i (H_m)_{jk}A^j \wedge A^i \wedge d\widetilde{B}\,.
\end{aligned} \tag{2.28}$$

The exterior differentiation of the equations (2.28) gives the second-order equations (2.14) and (2.21). If we take the exterior derivative of the last equation in (2.28) we have to make use of the identities (2.6), (2.7) and (2.8), also the fact that as $H_i^T\eta = -\eta H_i$ and since η is a diagonal matrix $H_i\eta$ must have anti-symmetric matrix representatives, to obtain the corresponding second-order equation in (2.21). We have also used the first-order equations (2.22) to derive (2.28). In the second equation above we have used the euclidean signature metric to raise the indices.

3. Dualisation

In this section we will dualize the bosonic field content of the $D = 7$ abelian Yang-Mills supergravity and we will construct the dualized coset element to realize the bosonic field equations namely (2.14) and (2.21) by means of the Cartan form of the dualized coset element. Our formulation will be in parallel with the one given in [11]. We will determine the structure constants of the Lie superalgebra which parameterize the dualized coset element so that the second-order bosonic field equations (2.14) and (2.21) can be obtained from the Cartan-Maurer equation which the Cartan form of the dualized coset element satisfies. Likewise in [11] we will also obtain the locally integrated first-order field equations (2.28) as a twisted self-duality equation of the Cartan form corresponding to the dualized coset element.

We start by assigning a generator for each bosonic field in (2.2). The original generators $\{K, V_i, Y, H_j, E_m\}$ will couple to the fields $\{\sigma, A^i, B, \phi^j, \chi^m\}$ respectively. The dual generators will also be introduced as $\{\widetilde{K}, \widetilde{V}_i, \widetilde{Y}, \widetilde{H}_j, \widetilde{E}_m\}$ which will be coupled to the dual fields $\{\widetilde{\sigma}, \widetilde{A}^i, \widetilde{B}, \widetilde{\phi}^j, \widetilde{\chi}^m\}$. We remind the reader of the fact that these dual fields have already appeared in the first-order field equations in (2.28) as a result of the local integration of the second-order field equations (2.14) and (2.21), they correspond to the Lagrange multiplier fields in our dualisation construction which is another manifestation of the Lagrange multiplier method which leads to the first-order formulation of the theory [22]. We require that the Lie superalgebra to be constructed from the original and the dual generators has the $\mathbb{Z}_2$ grading so that the generators will be chosen as odd if the corresponding potential is an odd degree differential form and otherwise even [11]. In particular $\{V_i, \widetilde{K}, \widetilde{Y}, \widetilde{H}_j, \widetilde{E}_m\}$ are odd generators and $\{K, \widetilde{V}_i, Y, H_j, E_m\}$ are even. As it will be clear later, the dualized coset element will be parameterized by a differential graded algebra which is generated by the differential forms and the generators we have introduced above. This algebra covers the Lie superalgebra of the field generators. The odd (even) generators behave like odd (even) degree differential forms under this graded differential algebra structure when they commute with the differential forms. The odd generators obey the anti-commutation relations while the even ones and the mixed couples obey the commutation relations.

Now let us consider the coset element

$$\nu = e^{\frac{1}{2}\phi^j H_j} e^{\chi^m E_m} e^{\sigma K} e^{A^i V_i} e^{\frac{1}{2}BY} . \tag{3.1}$$

The Cartan form

$$\mathcal{G} = d\nu\nu^{-1} , \tag{3.2}$$

which is a Lie superalgebra valued one-form can be calculated as

$$\begin{aligned}\mathcal{G} = {} & \frac{1}{2} d\phi^i H_i + \vec{\mathbf{E}}' \, \mathbf{\Omega} \, \vec{d\chi} + d\sigma K + \\ & + e^{c\sigma} \vec{\mathbf{V}} e^{\mathbf{U}} e^{\mathbf{B}} \vec{\mathbf{dA}} + OY ,\end{aligned} \tag{3.3}$$

where the three-form O is defined as

$$O = \frac{1}{2} e^{a\sigma} e^{\chi^m z_m} e^{\frac{1}{2}\phi^n v_n} (dB + A^i \wedge dA^j b_{ij}) . \tag{3.4}$$

We have defined the yet unknown structure constants as

$$
\begin{aligned}
[K,Y] &= aY\,, &\qquad \{V_i,V_j\} &= b_{ij}Y\,,\\
[H_i,Y] &= v_iY\,, &\qquad [E_m,Y] &= z_mY\,.
\end{aligned}
\tag{3.5}
$$

The matrices **U** and **B** are

$$
(\mathbf{U})^n_v = \frac{1}{2}\phi^i\theta^n_{iv}\,, \qquad (\mathbf{B})^j_n = \chi^m\beta^j_{mn}, \tag{3.6}
$$

where we have introduced the unknown structure constants as

$$
[H_i,V_n] = \theta^t_{in}V_t\,, \qquad [E_m,V_j] = \beta^l_{mj}V_l\,. \tag{3.7}
$$

We have also defined the commutators

$$
[K,V_i] = cV_i\,. \tag{3.8}
$$

In (3.3) the row vector $\vec{\mathbf{V}}$ is defined as (V_i) and the column vector $\vec{\mathbf{dA}}$ is (dA^i). In the derivation of (3.3) we have effectively used the formulas

$$
\begin{aligned}
de^Xe^{-X} &= dX + \frac{1}{2!}[X,dX] + \frac{1}{3!}[X,[X,dX]] + \cdots\,,\\
e^XYe^{-X} &= Y + [X,Y] + \frac{1}{2!}[X,[X,Y]] + \cdots\,.
\end{aligned}
\tag{3.9}
$$

We will now define the dualized coset element as

$$
\nu' = e^{\frac{1}{2}\phi^jH_j}e^{\chi^mE_m}e^{\sigma K}e^{A^iV_i}e^{\frac{1}{2}BY}e^{\frac{1}{2}\widetilde{B}\widetilde{Y}}e^{\widetilde{A}^i\widetilde{V}_i}e^{\widetilde{\sigma}\widetilde{K}}e^{\widetilde{\chi}^m\widetilde{E}_m}e^{\frac{1}{2}\widetilde{\phi}^j\widetilde{H}_j}\,. \tag{3.10}
$$

If we define the Cartan form $\mathcal{G}' = d\nu'\nu'^{-1}$ then it satisfies the Cartan-Maurer equation

$$
d\mathcal{G}' - \mathcal{G}'\wedge\mathcal{G}' = 0\,. \tag{3.11}
$$

As it is clear from the dualisation of the maximal supergravities in [11] one can determine the structure constants of the Lie superalgebra which generates the dualized coset element (3.10) either by calculating (3.11) and then comparing it with the second-order field equations (2.14) and (2.21) or one can directly calculate the Cartan form $\mathcal{G}'$ in terms of the unknown structure constants and then compare the twisted self-duality equation

$$
*\mathcal{G}' = \mathcal{S}\mathcal{G}'\,, \tag{3.12}
$$

with the first-order equations (2.28). Here $\mathcal{S}$ is a pseudo-involution of the Lie superalgebra of the original and the dual generators. For the field content of the $D=7$ abelian Yang-Mills supergravity it can be defined as [11, 14]

$$
\begin{aligned}
\mathcal{S}Y &= \widetilde{Y}\,, &\quad \mathcal{S}\widetilde{Y} &= -Y\,, &\quad \mathcal{S}E_m &= \widetilde{E}_m\,, &\quad \mathcal{S}\widetilde{E}_m &= -E_m\,,\\
\mathcal{S}K &= \widetilde{K}\,, &\quad \mathcal{S}\widetilde{K} &= -K\,, &\quad \mathcal{S}H_i &= \widetilde{H}_i\,, &\quad \mathcal{S}\widetilde{H}_i &= -H_i\,,\\
\mathcal{S}V_i &= \widetilde{V}_i\,, &\quad \mathcal{S}\widetilde{V}_i &= -V_i\,.
\end{aligned}
\tag{3.13}
$$

In our derivation of the algebra structure we will first use the fact that the Cartan form $\mathcal{G}'$ obeys the twisted self-duality equation (3.12) thus by using the identities (3.9) it can be written as

$$\begin{aligned}\mathcal{G}' = \mathcal{G} - \frac{1}{2} * d\phi^i \widetilde{H}_i - e^{\frac{1}{2}\alpha_i\phi^i}\mathbf{\Omega}^\alpha_\beta * d\chi^\beta \widetilde{E}_\alpha - \\ - *d\sigma\widetilde{K} - e^{c\sigma}\vec{\widetilde{\mathbf{V}}} e^{\mathbf{U}} e^{\mathbf{B}} * \vec{\mathbf{dA}} - *O\widetilde{Y}\,,\end{aligned} \qquad (3.14)$$

where we have defined the row vector $\vec{\widetilde{\mathbf{V}}}$ as $(\widetilde{\mathbf{V}}_i)$. In writing (3.14) we have also used the fact that the set of the original generators O and the set of the dual generators $\widetilde{D}$ obey the general scheme [11, 16, 17]

$$\begin{aligned} [O, \widetilde{D}\} \subset \widetilde{D}\,, \qquad [O, O\} \subset O\,, \\ [\widetilde{D}, \widetilde{D}\} = 0\,. \end{aligned} \qquad (3.15)$$

If we insert the Cartan form (3.14) into the Cartan-Maurer equation (3.11) and then compare the result with the second-order field equations (2.14) and (2.21) we can determine the desired structure constants of the original and the dual field generators. The calculation yields

$$\begin{aligned}
&[K, V_i] = \frac{1}{2}V_i\,, && [K, Y] = Y\,, && [K, \widetilde{Y}] = -\widetilde{Y}\,, \\
&[\widetilde{V}_k, K] = \frac{1}{2}\widetilde{V}_k\,, && \{V_i, V_j\} = -\frac{1}{\sqrt{2}}\eta_{ij}Y\,, && [H_l, V_i] = (H_l)^k_i V_k\,, \\
&[E_m, V_i] = (E_m)^j_i V_j\,, && [V_l, \widetilde{V}_k] = -\frac{2}{5}\delta_{lk}\widetilde{K} + \frac{1}{2}\sum_i (H_i)_{lk}\widetilde{H}_i\,, && \\
&\{V_k, \widetilde{Y}\} = 2\sqrt{2}\,\eta^l_k\,\widetilde{V}_l\,, && [Y, \widetilde{Y}] = \frac{16}{5}\widetilde{K}\,, && [H_i, \widetilde{V}_k] = -(H^T_i)^m_k \widetilde{V}_m\,, \\
&[E_\alpha, \widetilde{V}_k] = -(E^T_\alpha)^m_k \widetilde{V}_m\,, && && \\
&[H_j, E_\alpha] = \alpha_j E_\alpha\,, && [E_\alpha, E_\beta] = N_{\alpha,\beta} E_{\alpha+\beta}\,, && \\
&[H_j, \widetilde{E}_\alpha] = -\alpha_j \widetilde{E}_\alpha\,, && [E_\alpha, \widetilde{E}_\alpha] = \frac{1}{4}\sum_{j=1}^r \alpha_j \widetilde{H}_j\,, && \\
&[E_\alpha, \widetilde{E}_\beta] = N_{\alpha,-\beta}\widetilde{E}_\gamma\,, && \alpha - \beta = -\gamma,\ \alpha \neq \beta\,, &&
\end{aligned} \qquad (3.16)$$

where we have also included the commutation relations of the generators of $\mathbf{s}_0$. The matrices $((H_m)^j_i,\ (E_\alpha)^j_i)$ are the matrix representatives of the corresponding generators (H_m, E_α). Also the matrices $((H^T_m)^j_i,\ (E^T_\alpha)^j_i)$ are the matrix transpose of $((H_m)^j_i,\ (E_\alpha)^j_i)$. The scalar generators and their duals which are coupled to the five-form dual fields $\widetilde{\phi}^j$ and $\widetilde{\chi}^m$ namely the generators $\{H_i, E_m, \widetilde{E}_m, \widetilde{H}_i\}$ constitute a subalgebra and their algebra is already constructed for a generic scalar coset G/K in [15]. The commutators and the anti-commutators which are not listed in (3.16) vanish. Now we can explicitly calculate the Cartan form $\mathcal{G}'$ since we have obtained the algebra structure which generates the coset element ν'. Thus using the identities (3.9), the commutators and the anti-commutators of (3.16) the calculation of the dualized Cartan form yields

$$
\begin{aligned}
\mathcal{G}' &= d\nu'\nu'^{-1}\\
&= \frac{1}{2}d\phi^i H_i + e^{\frac{1}{2}\alpha_i\phi^i}U^\alpha E_\alpha + d\sigma K + e^{\frac{1}{2}\sigma}L^k_i dA^i V_k + \frac{1}{2}e^\sigma GY+\\
&\quad + \frac{1}{2}e^{-\sigma}d\widetilde{B}\widetilde{Y} + \left(\frac{4}{5}B\wedge d\widetilde{B} + \frac{2}{5}A^j\wedge d\widetilde{A}^i\delta_{ij} + d\widetilde{\sigma}\right)\widetilde{K}+\\
&\quad + \sum_{m=1}^{r}\Bigg((e^{\mathbf{\Gamma}}e^{\mathbf{\Lambda}})^m_j S^j - \frac{1}{2}(H_m)_{ji}A^j\wedge d\widetilde{A}^i -\\
&\qquad\qquad - \frac{1}{2\sqrt{2}}\eta^k_i(H_m)_{jk}A^j\wedge A^i\wedge d\widetilde{B}\Bigg)\widetilde{H}_m + \sum_{\alpha\in\Delta^+_{nc}}(e^{\mathbf{\Gamma}}e^{\mathbf{\Lambda}})^{\alpha+r}_j S^j\widetilde{E}_\alpha+\\
&\quad + \left(e^{-\frac{1}{2}\sigma}((L^T)^{-1})^l_k d\widetilde{A}^k + \sqrt{2}\,e^{-\frac{1}{2}\sigma}((L^T)^{-1})^l_k\,\eta^k_i\,A^i\wedge d\widetilde{B}\right)\widetilde{V}_l\,. \qquad (3.17)
\end{aligned}
$$

If we apply the twisted self-duality equation (3.12) on (3.17) we can find the locally integrated first-order field equations as

$$
\begin{aligned}
\frac{1}{2}e^\sigma * G &= -\frac{1}{2}e^{-\sigma}d\widetilde{B}\,,\\
e^{\frac{1}{2}\sigma}\mathrm{L}^i_j * dA^j &= -e^{-\frac{1}{2}\sigma}((L^T)^{-1})^i_j d\widetilde{A}^j - \sqrt{2}\,e^{-\frac{1}{2}\sigma}((L^T)^{-1})^i_j\,\eta^j_k\,A^k\wedge d\widetilde{B}\,,\\
*d\sigma &= -d\widetilde{\sigma} - \frac{4}{5}B\wedge d\widetilde{B} - \frac{2}{5}\delta_{ij}A^j\wedge d\widetilde{A}^i\,,\\
e^{\frac{1}{2}\alpha_i\phi^i}(\mathbf{\Omega})^\alpha_l * d\chi^l &= -(e^{\mathbf{\Gamma}}e^{\mathbf{\Lambda}})^{\alpha+r}_j S^j\,, \qquad (3.18)\\
\frac{1}{2} * d\phi^m &= -(e^{\mathbf{\Gamma}}e^{\mathbf{\Lambda}})^m_j S^j + \frac{1}{2}(H_m)_{ji}A^j\wedge d\widetilde{A}^i + \frac{1}{2\sqrt{2}}\eta^k_i(H_m)_{jk}A^j\wedge A^i\wedge d\widetilde{B}\,.
\end{aligned}
$$

These equations are the same equations with the first-order equations (2.28). Thus as we have intended we have obtained the locally integrated first-order equations (2.28) through the twisted self-duality equation (3.12) which the Cartan form $\mathcal{G}' = d\nu'\nu'^{-1}$ satisfies. This result also verifies the validity of the algebra structure derived in (3.16) which generates the dualized coset element (3.10) that yields the second-order field equations (2.14) and (2.21) in the Cartan-Maurer equation (3.11).

4. Conclusion

We have performed the bosonic dualisation of the $D = 7$ abelian Yang-Mills supergravity [21] which is the low energy effective limit of the fully Higgsed $D = 7$ heterotic string. We have discussed the coset construction of the scalars which belong to the coupling vector multiplets in section two. After deriving both the second and the first-order field equations in section two we have generalized the coset structure of the scalars to the entire bosonic sector by dualizing the original bosonic fields and then by determining the Lie superalgebra which generates the dualized coset element in section three. Therefore we have reformulated the bosonic sector of the $D = 7$ matter coupled supergravity as a non-linear sigma model. We have also obtained the first-order formulation of the bosonic sector of the $D = 7$ abelian Yang-Mills supergravity in the form of a twisted self-duality equation.

The dualisation method we have used in this work which is another manifestation of the Lagrange multiplier methods [22] is an extension of the one introduced in [11] to the matter coupled supergravities. Since the scalar coset manifold $\mathrm{SO}(N,3)/\mathrm{SO}(N)\times\mathrm{SO}(3)$ we are dealing with in this work is in general a non-split type depending on the number of coupling multiplets we have used the results of [15] which presents the dualisation of a generic non-split scalar coset. Thus this work also serves as an application of the achievements of [15].

Although we have derived the algebra which can be considered as a gauge to generate the dualized coset element, so that it realizes the bosonic field equations within the context of a non-linear sigma model, the group theoretical construction of the coset is to be studied. Furthermore one can also construct the dualized lagrangian and then determine the global and the local symmetry groups.

The symmetries of the supergravity theories have been studied in recent years to gain insight in the symmetries and the duality transformations of the string theories. Especially the global symmetries of the supergravities contribute to the knowledge of the non-perturbative U-duality symmetries of the string theories and the M theory [27, 28]. An appropriate restriction of the global symmetry group G of the effective low energy limit supergravity theory to the integers $\mathbb{Z}$, namely $G(\mathbb{Z})$, is conjectured to be the U-duality symmetry of the relative string theory [27]. The Lie superalgebra we have constructed in section three which generates the dualized coset element is a parametrization of the coset structure G/K of the bosonic sector thus it contains the necessary information about the enlarged symmetry group G of the $D=7$ matter coupled supergravity. Therefore as we have mentioned above the improved global symmetry analysis of the $D=7$ matter coupled supergravity may also reveal the symmetry scheme of the $D=7$ heterotic string.

As we have discussed in section two the T^3-compactification of the $D=10$ type-I supergravity that is coupled to the Yang-Mills theory [8, 9] gives the $D=7$ matter coupled supergravity, also an equivalent dual bosonic lagrangian of the $D=7$ abelian Yang-Mills supergravity in which the two-form potential B is replaced by a dual three-form field [18, 19] corresponds to the K_3 compactification [26] of the $D=11$ supergravity which is conjectured to be the low energy limit of the M theory. Thus by constructing a tool to study the general symmetries of the $D=7$ abelian Yang-Mills supergravity we have also contributed to the symmetry studies of these higher dimensional theories especially the M theory. In [20] the string-membrane dualities in $D=7$ which arise from the comparison of the construction of the $D=7$ abelian Yang-Mills supergravity either as a toroidally compactified heterotic string or a K_3-compactified $D=11$ supermembrane are discussed. It is possible that the complete dualisation and the non-linear realization of the bosonic sector of the $D=7$ abelian Yang-Mills supergravity will also help to understand the string-membrane and the string-string dualities in $D=7$.

One can also extend the construction of this work by including the gravity sector as well, in a way presented in [29, 30, 31] and then seek for the interpretation of the gravity-included dualized coset as a non-linear realization of a Kac-Moody global symmetry group.

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